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THEORETICAL AND EXPERIMENTAL INVESTIGATION OF SOLID-STATE MECHANISMS FOR GENERATING COHERENT RADIATION IN THE ULTRAVIOLET AND X-RAY REGIONS

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I. INTRODUCTION

The initial proposal for a coherent X-ray generator was based on double-stream interaction, which has been verified experimentally for electron beams, and which has been proposed and discussed a great deal for semiconductor plasmas. In the course of the investigation of the possibility of using double-stream interaction at X-ray frequencies, it became apparent that the excitation of bound electrons in the solids was a major complication. The complication results because at X-ray frequencies, $\hbar\omega$, the energy of a photon, is great enough to excite the bound electrons. Consequently, it was decided to investigate double-stream interaction at infrared frequencies, where $\hbar\omega$ is low enough that the bound electrons are not excited. This would allow verification of the basic interaction without the added complication of the bound electron excitations. Then the more complicated problem could be approached with much greater insight.

After the detailed investigation of the double-stream interaction was undertaken and an attempt was made to design and construct an infrared generator, it soon became apparent that there was a significant lack of knowledge about the double-stream interaction in solids. In fact, there seems to be no conclusive experimental verification of space-charge wave propagation (upon which double-stream interaction is based) in the high microwave or infrared frequencies. The Fifth Semiannual Report comprises the results of our detailed investigation of the double-stream interaction in solids at microwave and infrared frequencies. Since a major phase of the work was concluded and written up in the Fifth Semiannual Report, it seemed best not to include the lengthy details of the work again in this final report. The conclusion reached in the Fifth Semiannual Report was that the effects of thermal velocities, collisions, transverse ac velocities, and coupling problems combine to reduce the gain in double-stream interaction in solids so much that there seems to be little hope for utilizing that interaction without an applied dc magnetic field. Since a dc magnetic field would have to be impractically large at high microwave and infrared frequencies, double-stream interaction does not appear at all promising at these frequencies, and it is even less promising at X-ray frequencies. However, this is all theoretical work, and even though it is based on a rather realistic model, only experimental work can really indicate whether the results predicted by the analysis are valid. Consequently, experimental investigation of space-charge wave phenomena in solids is badly needed, and since this is noticeably lacking in the literature, we felt that the next logical step in our investigation should be such an experimental investigation.

The reason for the lack of experimental evidence of space-charge wave phenomena is that the measurement of space-charge wave phenomena is difficult for two main reasons: the wavelength of the space-charge waves is very short, making coupling very difficult, and the attenuation due to thermal velocities and collisions is very high, making propagation measurements very difficult. With the high attenuation, it is difficult to get a signal through a long enough sample that a difference in phase can be measured.

In this report we lay the groundwork for a method of investigating space-charge wave phenomena in solids which we hope will overcome the experimental difficulties and provide a great deal of information. The method consists of measuring the microwave impedance of very short semiconductor samples mounted in wave-guide circuits. Using short samples significantly reduces both the attenuation and the coupling problems. The microwave impedance concepts are discussed in Section II, and calculations of the impedance of short samples with one and two kinds of conduction charges are presented in Section III. These impedance expressions show how space-charge waves would effect the impedance and therefore could be detected.

II. IMPEDANCE CONCEPTS

2.1. Definition of Impedance

Impedance concepts at microwave frequencies are complicated because unique voltage and currents cannot always be defined. There are, however, special situations in which meaningful voltages and currents can be defined, and sometimes an impedance can be defined in terms of voltage and power, or current and voltage.

Let us discuss a cylindrical semiconductor plasma with perfectly conducting planes at both ends as shown in Fig. 1.

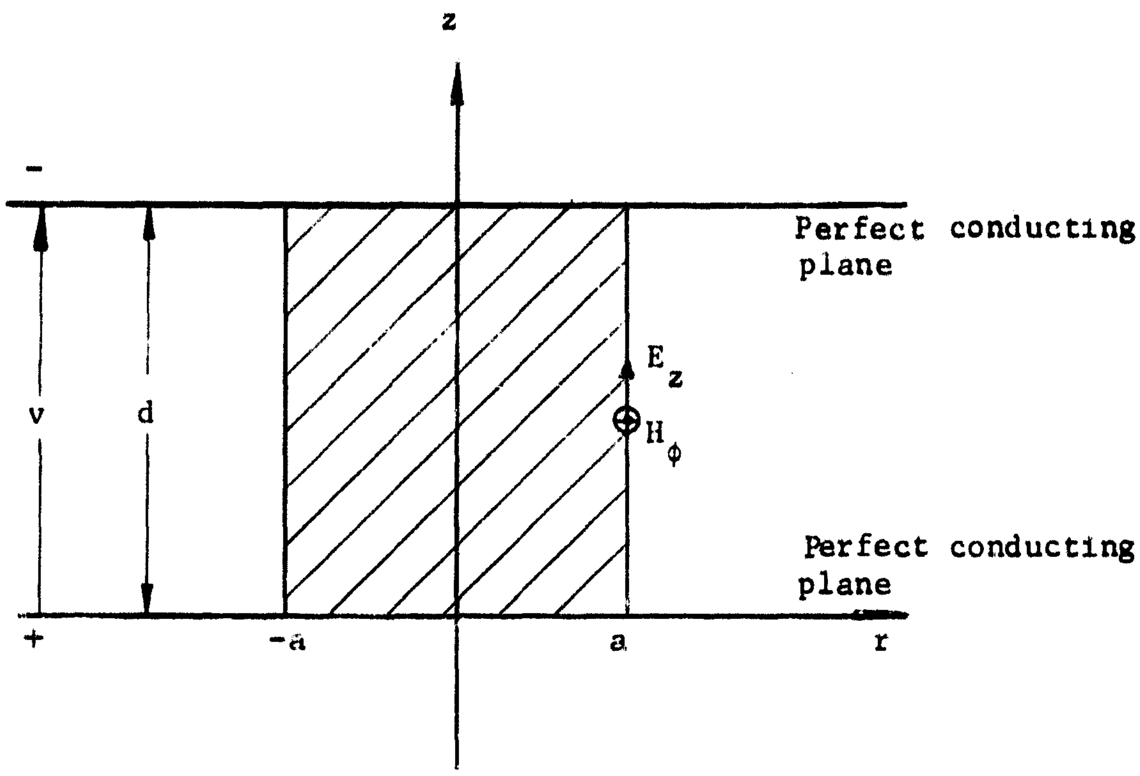

Fig. 1. A cylindrical semiconductor plasma with perfectly conducting planes at both ends.

It is assumed that $\frac{\partial}{\partial r} = \frac{\partial}{\partial \phi} = 0$, and d is extremely small compared to a wavelength. Then the semiconductor can be regarded as a point source to an external circuit. Therefore a meaningful voltage can be defined as

$$V = \int_0^d E(z)dz \tag{1}$$

A meaningful current can be defined as

$$I_T = \oint \vec{H} \cdot d\vec{\ell} = \int_0^{2\pi} H_\phi \, a \, d\phi = 2\pi \, a \, H_\phi \tag{2}$$

Since

$$\nabla \times \vec{H} = \vec{J} + j\omega\epsilon\vec{E} = \vec{J}_T = \text{constant} \tag{3}$$

$$I_T = J_T \quad \pi a^2 \tag{4}$$

Thus

$$H_\phi = \frac{J_T a}{2} \tag{5}$$

The impedance is often simply defined as $Z = V/I_T$, which fits with the idea of low-frequency circuit analysis. However, from the microwave point of view, which is more appropriate here, there can be no ac currents in the perfect conductors, and hence no power can be transmitted through the conductors. The only alternative is that the microwave power be transmitted radially through the $r = a$ boundary to the external circuit. This does not fit well with circuit theory concepts, and makes the definition of impedance as $Z = V/I_T$ questionable.

In addition, in a three-dimensional analysis, I_T will in general be a function of z, which further complicates the definition of impedance, because an impedance which is a function of z would not be useful in this case. A meaningful impedance can be defined, however, in terms of voltage and power. To get the impedance in the normal sense (i.e., a resistance corresponds to a power loss), the power is taken to be the power in the -r direction; that is, the power passing into the diode through the $r = a$ boundary. Then the impedance is defined as

$$Z = V^2/P \tag{6}$$

Happily, it can be shown that this impedance is exactly the same as V/I_T for the one-dimensional case, in which case I_T is not a function of z. The power is given by

$$P = \int_S \vec{E} \times \vec{H} \cdot d\vec{S}$$

In the one-dimensional case, this reduces to

$$P = \int_0^d \int_0^{2\pi} E_z H_\phi a d\phi dz = 2\pi a \int_0^d E_z H_\phi dz$$

Using Eqs 5, 4, and 1,

$$P = \frac{2\pi a^2 J_T}{2} \int_0^d E_z dz = I_T V$$

Hence

$$Z = V^2/I_T V = V/I_T \tag{7}$$

Thus the definition of impedance given in Eq. 6 is a general definition which is valid for short samples and which satisfies the microwave field concepts in the general case and reduces to the ordinary circuit-theory kind of impedance for the one-dimensional case. We will use Eq. 6 in all our subsequent work.

The concept of impedance given by our definition is very important, because it provides a method of getting a good approximate solution to the very complicated problem of the coupling of the semiconductor sample to an external microwave circuit. The principal alternative to the impedance kind of analysis is writing infinite-series field solutions inside and outside of the semiconductor and matching the boundary conditions. This seems to be hopelessly complicated and not likely to provide insight into the solution of the problem.

2.2 Calculation Procedures

The procedures for calculating the impedance are summarized below.

1. Write down the equations of motion for carriers (electrons or holes): the Lorentz force equations, the continuity equation, the current definition, and Maxwell's equations.
2. From the above equations, get the dispersion equation from which the propagation constants can be calculated. Also, the wave quantities corresponding to each propagating wave can be calculated.
3. Obtain the amplitude ratios of each propagating wave by matching boundary conditions at the ends.
4. Calculate the voltage between the two metal plates by integrating the electric field from injecting plate to collecting plate.
5. Calculate the impedance defined by Eq. 7.

2.3 Measurement of Impedance

In the preceding section, we have studied how to calculate the impedance of semiconductor samples. Actually it is very difficult to measure directly. In this section, we propose a practical method to measure it.

We mount our tiny cylindrical sample in a metal post and put it inside the wave guide with one end shorted, as shown in Fig. 2.

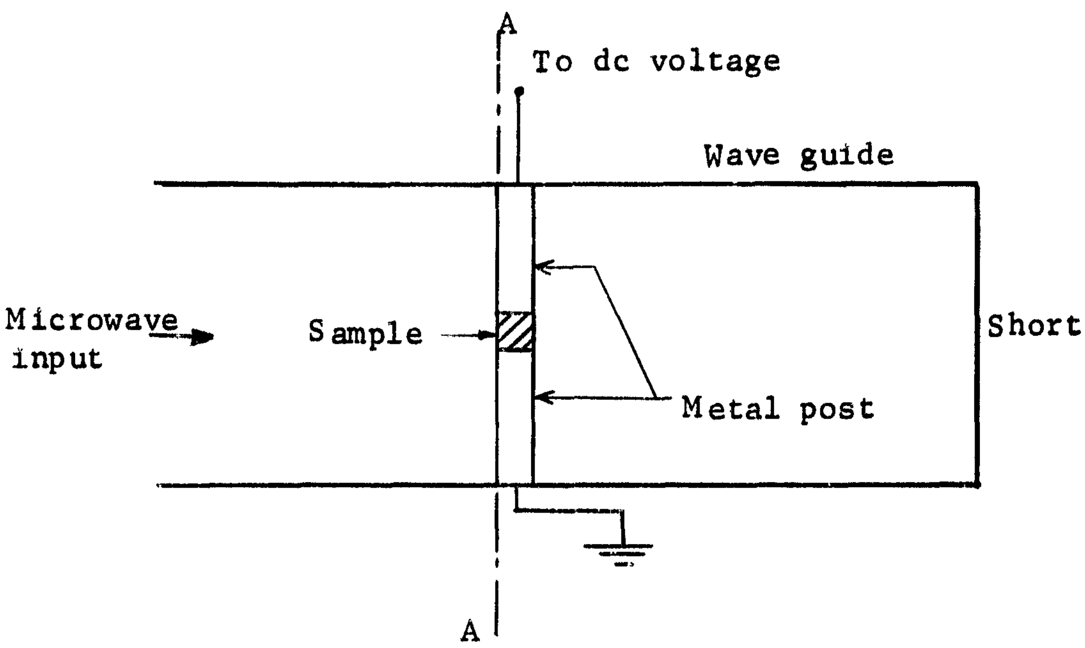

Fig. 2. Wave guide with sample.

For simplicity, we assume the post diameter is negligibly small. Then the equivalent circuit of Fig. 2 can be drawn as shown in Fig. 3.

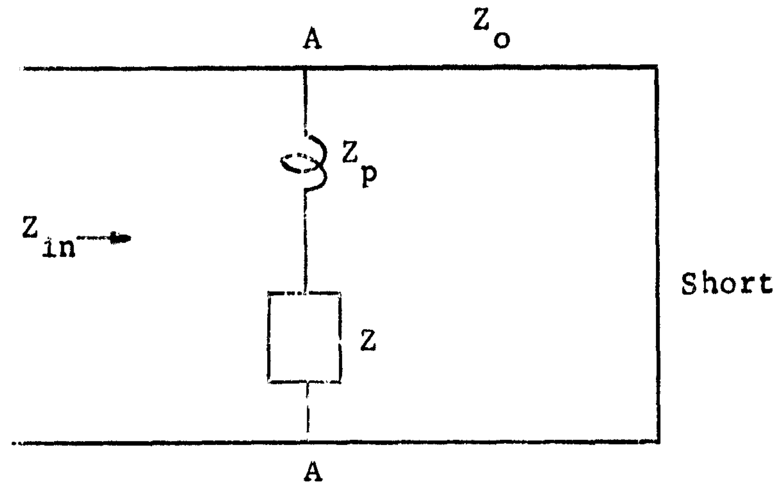

Fig. 3. Equivalent circuit of Fig. 2 where Z_o is the impedance of the waveguide. Z_p is the post inductance and Z is sample impedance.

The impedance looking from section A-A to the right can be drawn as shown in Fig. 4.

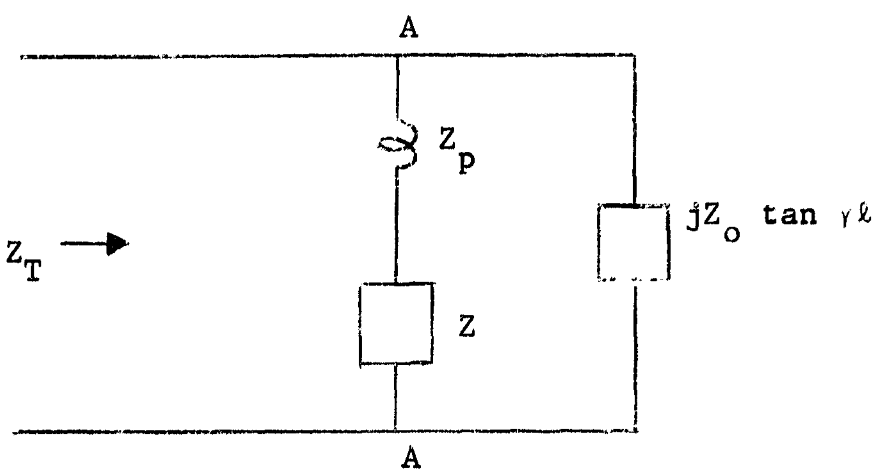

Fig. 4. Equivalent circuit of the impedance looking from section A-A where Z_T is the impedance looking from section A-A.

Then

$$Z_T = \frac{jZ_o \tan \gamma\ell \left(Z_p + Z\right)}{jZ_o \tan \gamma\ell + Z_p + Z} \tag{8}$$

Using the standing wave ratio, we are able to measure Z_T. And we also can measure Z_P. Finally we can calculate Z from the following equation.

$$\begin{aligned} Z &= \frac{jZ_o \tan \gamma\ell \left(Z_p - Z_T\right) - Z_T Z_p}{Z_T - jZ_o \tan \gamma\ell} \\ &= -\frac{jZ_o Z_T \tan \gamma\ell}{Z_T - jZ_o \tan \gamma\ell} - Z_p \end{aligned} \tag{9}$$

where γ is the propagation constant in the wave guide.

III. CALCULATION OF IMPEDANCE

Since we have studied how to calculate the impedance, we will analyze the impedance of both one-stream and two-stream cases in the following sections. Furthermore, we will discuss the boundary conditions, which are extremely important to our impedance calculation.

3.1 Impedance of One-Stream Interaction with Collision and Diffusion Effects

We assume that we have one-dimensional variation, time and space variation of $e^{j(\omega t - \beta z)}$, only one carrier (holes) present, and we make the small signal approximation. The basic equations for wave propagation in a semiconductor with a dc voltage in the z-direction follow.

The definition of ac current density is

$$J = \rho_o v + \rho u_o \tag{10}$$

The Lorentz force equation is

$$\left(\frac{\partial}{\partial t} + u_o \frac{\partial}{\partial z}\right) v + \frac{1}{\tau} v = \eta E - \frac{kT}{2m} \frac{\nabla \rho}{\rho_o} \tag{11}$$

The continuity equation is

$$\frac{\partial}{\partial t} J + \frac{\partial \rho}{\partial t} = 0 \tag{12}$$

From taking the divergence of Maxwell's curl H equation we get

$$\frac{\partial}{\partial z} (J + j\omega\varepsilon E) = 0 \tag{13}$$

where

J = ac current density

ρ_o, ρ = dc and ac charge density

$\eta = + |e|/m_h$

τ = collision time

$\frac{kT}{2m} = v_t^2$ = thermal velocity

Substituting Eq. 12, $\rho = \frac{\beta}{\omega} J$, into Eqs. 10 and 11, we have

$$\frac{j\, v_t^2 \beta^2}{\omega \rho_o} J + \left(j\omega - j\beta u_o + \frac{1}{\tau} \right) v - \eta E = 0 \tag{14}$$

$$\left(1 - \frac{\beta u_o}{\omega} \right) J - \rho_o v = 0 \tag{15}$$

and Eq. 15 can be rewritten as

$$\beta J + j\omega\epsilon\beta\, E = 0 \tag{16}$$

The dispersion equation can be obtained by eliminating J, v and E.

$$\begin{vmatrix} j \dfrac{v_t^2 \beta^2}{\omega \rho_o} & j\omega - j\beta u_o + \dfrac{1}{\tau} & -\eta \\ 1 - \dfrac{\beta u_o}{\omega} & -\rho_o & 0 \\ \beta & 0 & j\omega\epsilon\beta \end{vmatrix} = 0 \tag{17}$$

Expanding Eq. 17, we get

$$\beta \left[\left(\omega - \beta u_o \right) \left(\omega - \beta u_o - j\, \frac{1}{\tau} \right) + \beta^2 v_t^2 - \omega_p^2 \right] = 0 \tag{18}$$

Solving Eq. 18, we have

$$\beta = \beta_o = 0 \quad \text{and} \quad \beta = \beta_1 \quad \text{and} \ \beta_2, \tag{19}$$

where β_1 and β_2 are the roots of

$$\left(\omega - \beta u_o\right)\left(\omega - \beta u_o - j\frac{1}{\tau}\right) + \beta^2 v_t^2 - \omega_p^2 = 0$$

Substituting Eq. 19 into Eqs 14, 15 and 16, we can obtain the corresponding relationship between J, v and E with respect to each β individually. Assuming

$$X = \begin{bmatrix} J \\ v \\ E \end{bmatrix}$$

Eqs. 14, 15, and 16 can be written as

$$A^{op} X = 0$$

Then det $A^{op} = 0$ gives us the dispersion equation. The complete solution of X can be written as

$$X = k_o X_o + k_1 X_1 e^{-j\beta_1 z} + k_2 X_2 e^{-j\beta_2 z}$$

where k_o, k_1, and k_2 are undetermined constants. X_o, X_1, and X_2 are eigenvectors corresponding to eigenvalues β_o, β_1, and β_2. When $\beta = \beta_o = 0$ Eqs. 14 and 15 become

$$\left(j\omega + \frac{1}{\tau}\right) v - \eta E = 0$$

$$J - \rho_o v = 0$$

and

$$X_o = \begin{bmatrix} \eta\rho_o \\ \eta \\ \left(j\omega + \frac{1}{\tau}\right) \end{bmatrix}$$

When $\beta = \beta_i$ $(i = 1, 2)$, Eqs. 14 and 15 become

$$j \frac{\beta_i^2 v_t^2}{\omega \rho_o} J + \left(j\omega - j\beta_i u_o + \frac{1}{\tau} \right) v - \eta E = 0$$

$$\left(1 - \frac{\beta_i u_o}{\omega} \right) J - \rho_o v = 0$$

So

$$X_i = \begin{bmatrix} \rho_o \\ 1 - \frac{\beta_i u_o}{\omega} \\ - \frac{\rho_o}{j\omega\varepsilon} \end{bmatrix} \quad (i = 1, 2) \tag{20}$$

Thus

$$X = k_o X_o + \sum_{i=1}^{2} k_i X_i e^{-j\beta_i z}$$

i.e.,

$$\begin{bmatrix} J \\ v \\ E \end{bmatrix} = k_o \begin{bmatrix} \eta \rho_o \\ \eta \\ j\omega + \frac{1}{\tau} \end{bmatrix} + \sum_{i=1}^{2} k_i \begin{bmatrix} \rho_o \\ 1 - \frac{\beta_i u_o}{\omega} \\ - \frac{\rho_o}{j\omega\varepsilon} \end{bmatrix} e^{-j\beta_i z} \tag{21}$$

Applying the boundary conditions[1] $J = 0$ at $z = 0$, $v = 0$ at $z = 0$, and $J + j\omega\varepsilon E = J_T$ = constant, we have the following results:

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[1] See next section.

$$k_o \, n\rho_o + k_1 \, \rho_o + k_2 \, \rho_o = 0 \tag{22}$$

$$k_o \, n + k_1 \left(1 - \frac{\beta_1 \, u_o}{\omega}\right) + k_2 \left(1 - \frac{\beta_2 \, u_o}{\omega}\right) = 0 \tag{23}$$

$$\begin{aligned} & k_o \, n\rho_o + \sum_{i=1}^{2} k_i \, \rho_o \, e^{-j\beta_i z} + j\omega\varepsilon \left[k_o \left(j\omega + \frac{1}{\tau}\right) \right. \\ & \qquad \left. + \sum_{i=1}^{2} k_i \left(- \frac{\rho_o}{j\omega\varepsilon}\right) e^{-j\beta_i z} \right] \\ & = k_o \, n\rho_o + k_o \, j\omega\varepsilon \left(j\omega + \frac{1}{\tau}\right) = J_T \end{aligned} \tag{24}$$

From Eqs. 22 and 23, we have

$$\frac{k_o}{\begin{vmatrix} 1 & 1 \\ \left(1 - \frac{\beta_1 u_o}{\omega}\right) & \left(1 - \frac{\beta_2 u_o}{\omega}\right) \end{vmatrix}} = \frac{k_1}{\begin{vmatrix} 1 & n \\ \left(1 - \frac{\beta_2 u_o}{\omega}\right) & n \end{vmatrix}} = \frac{k_2}{\begin{vmatrix} n & 1 \\ n & \left(1 - \frac{\beta_1 u_o}{\omega}\right) \end{vmatrix}}$$

From Eq. 24, we get

$$k_o = \frac{J_T}{n\rho_o + j\omega\varepsilon \left(j\omega + \frac{1}{\tau}\right)} \tag{25}$$

$$k_1 = \frac{n\beta_2}{\left(\beta_1 - \beta_2\right)} \tag{26}$$

$$k_2 = - \frac{n\beta_1}{\left(\beta_1 - \beta_2\right)} \tag{27}$$

Thus $E(z) = k_o\left(j\omega + \frac{1}{\tau}\right) + \sum_{i=1}^{2} k_i\left(-\frac{\rho_o}{j\omega\epsilon}\right)e^{-j\beta_1 z}$. The potential difference is given by

$$V = -\int_d^0 E\,dz = V_o - V_d$$

$$= \int_0^d E\,dz$$

$$= \int_0^d \left[k_o\left(j\omega + \frac{1}{\tau}\right) + \sum_{i=1}^{2} k_i\left(-\frac{\rho_o}{j\omega\epsilon}\right)e^{-j\beta_i z}\right]dz$$

$$= k_o\left(j\omega + \frac{1}{\tau}\right)d + \sum_{i=1}^{2} k_i\left(-\frac{\rho_o}{\omega\epsilon\beta_i}\right)\left(e^{-j\beta_i d} - 1\right) \tag{28}$$

The impedance Z is defined as $\frac{V}{JA}$, where A is a cross section of semiconductor.

$$Z = \frac{V}{JA} = \frac{1}{A\left[n\rho_o + j\omega\epsilon\left(j\omega + \frac{1}{\tau}\right)\right]}\left[\left(j\omega + \frac{1}{\tau}\right)d - \frac{n\rho_o\,\beta_2}{\omega\epsilon\beta_1\left(\beta_1 - \beta_2\right)}\left(e^{-j\beta_1 d} - 1\right) + \frac{n\rho_o\,\beta_1}{\omega\epsilon\beta_2\left(\beta_1 - \beta_2\right)}\left(e^{-j\beta_2 d} - 1\right)\right] \tag{29}$$

When $\rho_o \to 0$, $\tau \to \infty$, then

$$Z = \frac{1}{j\omega\epsilon A \cdot j\omega}\,[j\omega d] = \frac{d}{j\omega\epsilon A} = \frac{1}{j\omega C} \tag{30}$$

where $C = \epsilon\frac{A}{d}$. Thus z reduces to the capacitive impedance when no conduction charges are present, as it should. The impedance Z is not only a function of the propagation constant β_i, but is also a function of length d Because Eq. 29 is so complicated, it is difficult to tell how the impedance Z changes with frequency and other parameters without the computer's help. However, it can be seen qualitatively from Eq. 29 that the $\left(e^{-j\beta_1 d}-1\right)$ and $\left(e^{-j\beta_2 d}-1\right)$ terms result from space-charge waves, and these terms will affect the impedance significantly.

3.2 Boundary Conditions

The boundary conditions usually can be obtained by proper integration of Maxwell equations In the one-dimensional case shown in Fig. 5, the boundary conditions are $D_2 = \rho_s$, $J_2 + j\omega\varepsilon_2 E_2 = J_T$ = constant since only a longitudinal electric field is present in the one-dimensional case.

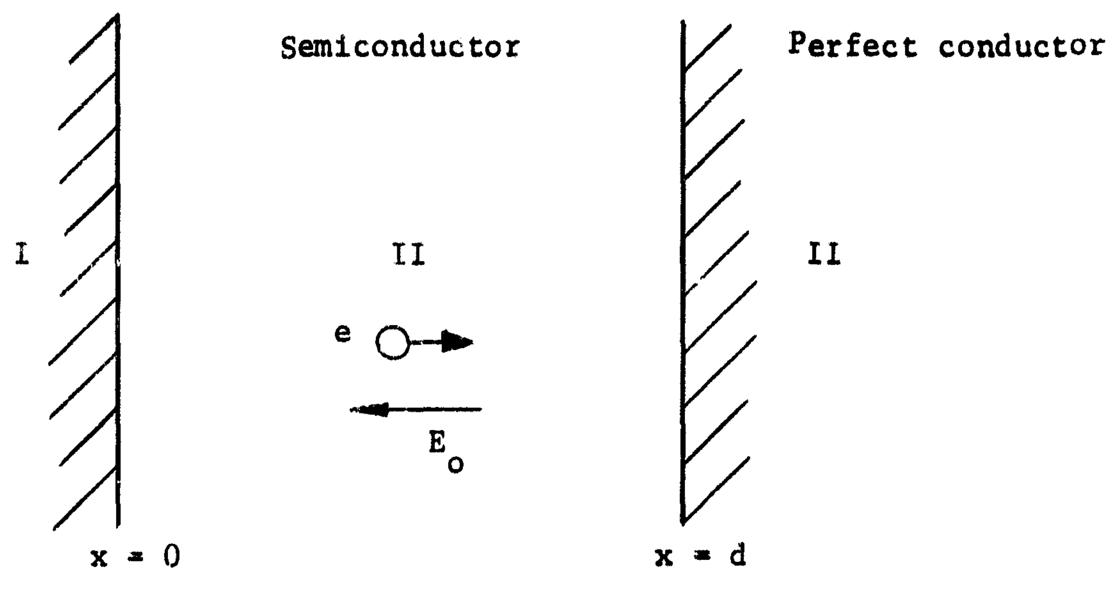

Fig. 5. Boundary conditions between a semiconductor and perfect metal contacts.

Unfortunately, these boundary conditions cannot give us enough information to solve the problem. Some other boundary conditions must be found or postulated. Van der Ziel[2] used boundary conditions that the ac electric field $E_2 = 0$ and the ac potential $\phi_2 = 0$ at the initial plate $x = 0$, and the ac potential $\phi_2 = \phi_a$ at the second plate, $x = d$, in the space-charge-limited, solid-state diode case. Kawamura[3] used the boundary conditions that the ac electric field E must be zero at both ends, $x = 0$ and $x = d$.

Van der Ziel's boundary conditions do not fit our problem because we are not considering the space-charge-limited case. Kawamura's boundary conditions appear to be incorrect. $E = 0$ at both ends implies no surface charge exists on the perfect conducting surface. We do not think that this is possible.

In our work, we shall use $J = 0$ and $v = 0$ at the initial plate, $x = 0$, as the boundary conditions. In the perfect conductor, the ac fields, currents and charge densities are all zero. The electrons which leave the perfect conductor and enter the semiconductor will be accelerated by the ac electric fields in the semiconductor, but since it takes a finite time for these electrons to acquire ac velocity, their ac velocities at the boundary must be zero.

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[2] A. Van der Ziel, S. T. Hsu, "High-Frequency Admittance of Space-Charge-Limited Solid-State Diodes," *Proc. IEEE*, Sept. 1966, p. 1194.

[3] M. Kawamura, S. Morishita, "A New Negative Resistance of Semiconductor Bulk," *Proc. IEEE*, July 1963, p. 1213.

3.3 Impedance of Double-Stream Interaction Without Collision and Diffusion Effects.

The double-stream model is shown in Fig. 6. Combining the definition of small-signal current density

$$J_i = \rho_{oi} v_o + \rho_i u_{oi} \qquad (i = 1, 2) \tag{31}$$

and the continuity equation,

$$\frac{\partial J_i}{\partial z} + \frac{\partial \rho_i}{\partial t} = 0 \tag{32}$$

$$\beta J_i - \omega \rho_i = 0 \tag{33}$$

we have

$$\left(j\omega - j\beta u_{oi}\right) J_i - j\omega \rho_{oi} v_i = 0 \qquad i = 1, 2 \tag{34}$$

where $i = 1, 2$ for the two streams.

From the Lorentz force equation, we have

$$\left(j\omega - j\beta u_{oi}\right) v_i - \eta_i E = 0 \qquad i = 1, 2 \tag{35}$$

From $\nabla \cdot \nabla \times \vec{H} = \nabla \cdot \left(\vec{J} + j\omega\varepsilon\vec{E}\right) = 0$, we have

$$\beta J_1 + \beta J_2 + j\beta\omega\varepsilon E = 0 \tag{36}$$

Eliminating J_1, J_2, v_1, v_2 and E from Eqs. 34, 35 and 36 we get the dispersion equation

$$\beta = 0 \tag{37}$$

and

$$\frac{\omega_{p1}^2}{\left(\omega - \beta u_{o1}\right)^2} + \frac{\omega_{p2}^2}{\left(\omega - \beta u_{o2}\right)^2} = 1 \tag{38}$$

where

$$\omega_{pi}^2 = \frac{\eta_i \rho_{oi}}{\varepsilon}$$

$$\eta_i = q_i/m_i$$

= ratio of charge to mass. (Can be + or -.)

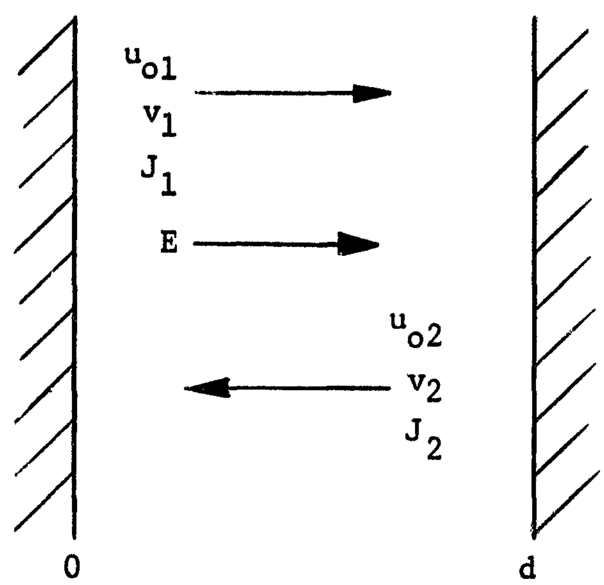

Fig. 6. Double-stream interaction in semiconductors. Subscript 1 stands for holes and subscript 2 stands for electrons.

From the dispersion relationship, we know there are five propagation constants, $\beta = \beta_o = 0$, and β_i[4] ($i = 1$ to 4) satisfying Eq. 38.

Let

$$X = \begin{bmatrix} J_1 \\ v_1 \\ J_2 \\ v_2 \\ E \end{bmatrix}$$

Then the complete solution can be written as

$$X = k_o X_o + \sum_{i=1}^{4} k_i X_i e^{-j\beta_i z} \tag{39}$$

When $\beta = \beta_o = 0$, Eq. 34 and 35 become

$$\left(j\omega - j\beta u_{oi}\right)J_i - j\omega\rho_{oi}v_i = 0$$

$$j\omega v_i - \eta_i E = 0$$

so

$$X_o = \begin{bmatrix} \rho_{o1}\eta_1 \\ \eta_1 \\ \rho_{o2}\eta_2 \\ \eta_2 \\ j\omega \end{bmatrix}$$

When $\beta = \beta_i$, Eqs. 34 and 35 become

---

[4] β_i is the root of Eq. 38 (fourth order). It can be solved by computer methods.

$$\left(j\omega - j\beta u_{oi}\right) J_i - j\omega\, \rho_{oi} v_i = 0$$

$$\left(j\omega - j\beta u_{oi}\right) v_i - \eta_i E = 0$$

Then

$$X_i = \begin{bmatrix} j\omega\varepsilon \dfrac{\omega_{pi}^2}{\left(\omega - \beta_i u_{o1}\right)^2} \\[2ex] \dfrac{j\eta_1}{\left(\omega - \beta_i u_{o1}\right)} \\[2ex] j\omega\varepsilon \dfrac{\omega_{p2}^2}{\left(\omega - \beta_i u_{o2}\right)^2} \\[2ex] \dfrac{j\eta_2}{\left(\omega - \beta_2 u_{o2}\right)} \end{bmatrix}$$

$$-1$$

Thus Eq. 39 turns out to be

$$X = \begin{bmatrix} J_1 \\ v_1 \\ J_2 \\ v_2 \\ E \end{bmatrix} = k_o \begin{bmatrix} \rho_{o1}\,\eta_1 \\ \eta_1 \\ \rho_{02}\,\eta_2 \\ \eta_2 \\ j\omega \end{bmatrix} + \sum_{i=1}^{4} k_i \begin{bmatrix} j\omega\varepsilon \dfrac{\omega_{p1}^2}{\left(\omega - \beta_i u_{o1}\right)^2} \\[2ex] \dfrac{j\eta_1}{\left(\omega - \beta_i u_{o1}\right)} \\[2ex] j\omega\varepsilon \dfrac{\omega_{p2}^2}{\left(\omega - \beta_i u_{o2}\right)^2} \\[2ex] \dfrac{j\eta_2}{\left(\omega - \beta_i u_{o2}\right)} \\[2ex] -1 \end{bmatrix} e^{-j\beta_i z} \qquad (40)$$

where k_o and k_i ($i = 1$ to 4) are undetermined constants. By matching the boundary conditions at both ends we can get the relationships between them. At $z = 0$, $J_1 = 0$, and $v_1 = 0$. This implies

$$k_o \rho_{o1} \eta_1 + \sum_i k_i \frac{j\omega\varepsilon\,\omega_{p1}^2}{\left(\omega - \beta_i u_{o1}\right)^2} = 0 \tag{41}$$

$$k_o \eta_1 + \sum_i k_i \frac{j\eta_1}{\left(\omega - \beta_i u_{o1}\right)} \tag{42}$$

At $z = d$, $J_2 = 0$ and $v_2 = 0$. This implies

$$k_o \rho_{o2} \eta_2 + \sum_i k_i \frac{j\omega\varepsilon\,\omega_{p2}^2}{\left(\omega - \beta_i u_{o2}\right)^2} e^{-j\beta_i d} = 0 \tag{43}$$

$$k_o \eta_2 + \sum_i k_i \frac{j\eta_2}{\left(\omega - \beta_i u_{o2}\right)} e^{-j\beta_i d} = 0 \tag{44}$$

$$k_i = k_o \frac{\Delta_i}{\Delta_o} \tag{45}$$

where

$$\Delta_o = \begin{vmatrix} \frac{j\omega}{\left(\omega - \beta_1 u_{o1}\right)^2} & \frac{j\omega}{\left(\omega - \beta_2 u_{o1}\right)^2} & \frac{j\omega}{\left(\omega - \beta_3 u_{o1}\right)^3} & \frac{j\omega}{\left(\omega - \beta_4 u_{o1}\right)^2} \\ \frac{j}{\left(\omega - \beta_1 u_{o1}\right)} & \frac{j}{\left(\omega - \beta_2 u_{o1}\right)} & \frac{j}{\left(\omega - \beta_3 u_{o1}\right)} & \frac{j}{\left(\omega - \beta_4 u_{o1}\right)} \\ \frac{j\omega e^{-j\beta_1 d}}{\left(\omega - \beta_1 u_{o2}\right)^2} & \frac{j\omega e^{-j\beta_2 d}}{\left(\omega - \beta_2 u_{o2}\right)^2} & \frac{j\omega e^{-j\beta_3 d}}{\left(\omega - \beta_3 u_{o2}\right)^2} & \frac{j\omega e^{-j\beta_4 d}}{\left(\omega - \beta_4 u_{o2}\right)^2} \\ \frac{j\, e^{-j\beta_1 d}}{\left(\omega - \beta_1 u_{o2}\right)} & \frac{j\, e^{-j\beta_2 d}}{\left(\omega - \beta_2 u_{o2}\right)} & \frac{j\, e^{-j\beta_3 d}}{\left(\omega - \beta_3 u_{o2}\right)} & \frac{j\, e^{-j\beta_4 d}}{\left(\omega - \beta_4 u_{o2}\right)} \end{vmatrix}$$

$\Delta_i = (-1)^i$ times the determinant which is Δ_o with the ith column replaced by 1.

From another boundary condition $J_1 + J_2 + j\omega\varepsilon E = J_T$. Thus we have

$$k_o \left[\rho_{o1}\eta_1 + \rho_{o2}\eta_2 + j\omega\varepsilon \cdot j\omega\right] = J_T$$

$$k_o = \frac{J_T}{\varepsilon\, \omega_{p1}^2 + \varepsilon\omega_{p2}^2 - \varepsilon\omega^2} \tag{48}$$

So the electric field is given by

$$E(z) = k_o \left[j\omega - \sum_{i=1}^{4} \frac{\Delta_i}{\Delta_o} e^{-j\beta_i z} \right] \tag{49}$$

$$V = V_o - V_d = -\int_d^0 E(z)\,dz$$

$$= \int_0^d E(z)\,dz$$

$$= \int_0^d k_o \left[j\omega - \sum_{i=1}^{4} \frac{\Delta_i}{\Delta_o} e^{-j\beta_i z} \right] dz$$

$$= k_o\ j\omega d + \sum_{i=1}^{4} \frac{\Delta_i}{\Delta_o} \cdot \frac{1}{+j\beta_i} \left[e^{-j\beta_i d} - 1 \right] \tag{50}$$

Thus

$$Z = \frac{V}{J_T A} = \frac{1}{A\varepsilon \left(\omega_{p1}^2 + \omega_{p2}^2 - \omega^2 \right)} \left\{ j\omega d + \sum_{i=1}^{4} \frac{\Delta_i}{\Delta_o} \frac{1}{j\beta_i} \left[e^{-j\beta_i d} - 1 \right] \right\} \tag{51}$$

If ω_{p1} and ω_{p2} approach 0, then $Z = \dfrac{1}{-\varepsilon\omega^2 A}\, j\omega d$

$$= \frac{1}{j\omega C}$$

where $C = \varepsilon \dfrac{A}{d}$. Thus Z reduces to the capacitive reactance, just as it should when no conduction charges are present.

Equation 51 shows that the impedance Z again is a complicated function. With further investigation, the characteristic ways in which the space-charge wave interactions affect Z can be determined, and impedance measurements can be made and correlated with the theoretical values to verify the nature of space-charge waves in solids.

IV. CONCLUSIONS

After a thorough investigation of double-stream interaction in solids, it is concluded that the practical generation of coherent X-rays by double-stream interaction is very unlikely The detailed reasons are given in the Fifth Semiannual Report, which concluded a major phase of the work. Briefly, the main problems are the losses in gain produced by thermal velocities, collisions, transverse ac velocities, and weak coupling.

Since there is a significant lack of experimental work in space-charge phenomena in solids, it is difficult to really know whether there are possibilities of using space-charge interactions in practical devices. We have concluded that a good way to overcome the experimental difficulties involved in measuring space-charge wave phenomena is the microwave impedance technique, which we have discussed in this report.